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MODELING OF THE ROTATIONAL MOTION OF A DISPERSED PHASE USING THE EQUATIONS OF TRANSFER OF THE SECOND AND THIRD MOMENTS OF PULSATIONS OF THE TRANSLATIONAL AND ANGULAR VELOCITIES OF PARTICLES

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This article considers the Eulerian continuum description of turbulent transfer of momentum and moment of momentum in a solid phase on the basis of the equations of transfer of the second and third moments of pulsations of the linear and angular velocities of particles. The pulsating characteristics of a gas are computed using the two-parameter model of turbulence generalized to the case of gas-dispersed turbulent flows.

At the present time, the problem of obtaining a closed system of equations of turbulent motion of a disperse flow on the basis of a rough approximation of unknown third-order correlations in the governing equations of transfer of the second moments of pulsations of the linear and angular velocities of particles has received much attention. Definite progress has been made in finding the third moments of pulsations of the translational velocity of particles in the absence of their rotation. In [1, 2], the sought values $\langle v'_p v'_p v'_p \rangle$, $\langle v'_p w'_p w'_p \rangle$, etc. were determined with the aid of algebraic relations that express third-order correlations in terms of the second moments and their gradients by solving truncated equations of transfer of the third moments. This made it possible to obtain a close description of the motion of a dispersed phase at the level of equations for second-order correlations. In [3, 4], to find the sought variables $\langle w'_p w'_p w'_p \rangle$ and $\langle u'_p w'_p w'_p \rangle$, the transfer equations of the variables themselves were used. Closing of the indicated equations was made on the basis of the algebraic relations obtained by solving the truncated equations for the fourth moments. Thus, a closed description of the motion of particles was obtained at the level of equations for triple correlations.

At the same time, modeling of the mixed third moments of pulsations of the linear and angular velocities of particles is restricted to gradient representations. Thus, for example, in [5], to find the unknown quantities $\langle \omega'_\varphi v'^2_p \rangle$, $\langle \omega'_\varphi w'^2_p \rangle$, $\langle \omega'_\varphi \omega'_r w'^2_p \rangle$, etc., the coefficient of turbulent viscosity of the "gas" of particles $\eta_{t,p}$ is introduced, which relates the third moments to the gradients of the second moments (for example, $\langle \omega'^2_\varphi v'_p \rangle = -\eta_{t,p}\, \partial \langle \omega'^2_\varphi \rangle / \partial r$).

It seems that in the present work an attempt has been made for the first time to obviate the need for gradient representations of mixed correlations by using the developed calculation technique [2], according to which a chain of axisymmetric averaged transfer equations of the second and third moments of pulsations of the translational and angular velocities of a dispersed phase over the stretch of stabilized ascending motion of a gas suspension was obtained with account for the interphase and interparticle interaction. The equations for triple correlations were closed by representating the fourth moments in the form of a sum of products of the second moments, thus making it possible to obtain a closed description of the motion of a dispersed phase at the level of equations for double correlations. In order to compute the pulsating characteristics of the carrying medium, a modified two-parameter model of turbulence is used which accounts for the influence of particles [6].

In constructing the system of differential equations of transfer of averaged and pulsating characteristics of a two-phase flow on the basis of interpenetrating continua, interacting among themselves, the following simplifying premises are used: 1) the process is stationary; 2) over the stretch of a stabilized gas-suspension flow there is no averaged radial and transversal motion of phases, and the averaged parameters do not change in the axial direction; 3) the vector of the angular velocity of particles $\boldsymbol{\omega}$ is directed along the transversal axis; 4) the rotational pulse of collisional origin is not taken into account [7]; 5) the solid phase concentration is uniformly distributed over the channel

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section; 6) pulsations of the angular velocity of the rotation of a liquid element are ignored, and 7) the solid phase consists of monodisperse spherical particles.

The system of equations that describes the behavior of a two-phase flow over the stretch of a steady motion of a gas suspension has the form

$$\frac{\rho_g}{r}\frac{\partial}{\partial r}\left[r\,(\eta_{t,g}+\eta_g)\frac{\partial u_g}{\partial r}\right]-\frac{\partial P}{\partial z}-F_{az}-F_{Mz}=0\,,\quad \frac{\rho_p\beta}{r}\frac{\partial}{\partial r}\left(r\,\langle u'_p v'_p\rangle\right)-F_{az}-F_{Mz}+\rho_p\beta g=0\,,$$

$$\langle u'_p v'_p\rangle=-\,\eta_{t,p}\partial u_p/\partial r\,,\quad F_{Mz}=\lambda_\omega\beta\rho_p\left(\langle\omega'_r w'_p\rangle-\langle\omega'_\varphi v'_p\rangle\right),\quad \lambda_\omega=3\rho_g/4\rho_p\,,\quad \eta_{t,p}=\chi\left(\langle v'^2_p\rangle\right)\ \ [5]\,, \tag{1}$$

$$\frac{\rho_g}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\eta_{t,g}}{\sigma_k}+\eta_g\right)\frac{\partial k_g}{\partial r}\right]+\rho_g\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2-\rho_g\,(\varepsilon_g+\varepsilon_p)+G=0\,, \tag{2}$$

$$\frac{\rho_g}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\eta_{t,g}}{\sigma_d}+\eta_g\right)\frac{\partial \varepsilon_g}{\partial r}\right]+\rho_g L_1\frac{\varepsilon_g}{k_g}\xi_1\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2-\rho_g L_2\frac{\varepsilon_g^2}{k_g}\xi_2-\rho_g S=0\,, \tag{3}$$

$$\frac{\partial\left(r\,\langle\omega'_\varphi v'_p\rangle\right)}{r\partial r}+\frac{\langle\omega'_r w'_p\rangle}{r}=-\,\gamma\left(\omega_\varphi+\frac{\partial u_g}{2\partial r}\right),\quad \gamma=\frac{60\rho_g\eta_g}{\rho_p\delta^2}\,. \tag{4}$$

The system of equations (1)–(4) is indeterminate, since the unknown variables $\langle\omega'_r w'_p\rangle$, $\langle\omega'_\varphi v'_p\rangle$, and $\langle v'^2_p\rangle$ are present in it. To derive the transfer equations of second-order correlations it is first of all necessary to obtain pulsational equations of momentum and moment of momentum of the dispersed phase. In [4], pulsational equations were constructed for the translational velocity of particles along the radial and transversal axes. With allowance made for the Magnus force F_M, these equations can be presented in the form

$$u_p\frac{\partial v'_p}{\partial z}+u'_p\frac{\partial v_p}{\partial z}+u'_p\frac{\partial v'_p}{\partial z}+v_p\frac{\partial v'_p}{\partial r}+v'_p\frac{\partial v_p}{\partial r}+v'_p\frac{\partial v'_p}{\partial r}-\frac{1}{r}w'_p w'_p-\frac{\partial\langle u'_p v'_p\rangle}{\partial z}-$$

$$-\frac{\partial\left(r\,\langle v'_p v'_p\rangle\right)}{r\partial r}+\frac{1}{r}\langle w'_p w'_p\rangle=\frac{F'_{ar}+F'_{Mr}}{\rho_p\beta}\,, \tag{5}$$

$$u_p\frac{\partial w'_p}{\partial z}+u'_p\frac{\partial w'_p}{\partial z}+v_p\frac{\partial w'_p}{\partial r}+v'_p\frac{\partial w'_p}{\partial r}+\frac{1}{r}\left(v_p w'_p+w'_p v'_p\right)-\frac{\partial\langle u'_p w'_p\rangle}{\partial z}-$$

$$-\frac{\partial\left(r\,\langle v'_p w'_p\rangle\right)}{r\partial r}-\frac{1}{r}\langle w'_p v'_p\rangle=\frac{F'_{a\varphi}+F'_{M\varphi}}{\rho_p\beta}\,, \tag{6}$$

where

$$F'_{ar}=\frac{\rho_p\beta}{\tau}\left(v'_g-v'_p\right);\ F'_{a\varphi}=\frac{\rho_p\beta}{\tau}\left(w'_g-w'_p\right);\ F'_{Mr}=-\,\lambda_\omega\rho_p\,\beta\,(u_g-u_p)\,\omega'_\varphi\,;\ F'_{M\varphi}=\lambda_\omega\rho_p\,\beta\,(u_g-u_p)\,\omega'_r\,. \tag{7}$$

Let us project the stationary actual equation of rotational motion of particles onto the radial and transversal coordinate axes. In view of the axial symmetry of the problem ($\partial/\partial\varphi = 0$) the projections of the indicated equation have the form [6]

$$\hat{u}_{\rm p}\frac{\partial\hat{\omega}_\varphi}{\partial z}+\hat{\upsilon}_{\rm p}\frac{\partial\hat{\omega}_\varphi}{\partial r}+\frac{\hat{\omega}_r\hat{w}_{\rm p}}{r}=-\gamma\left[\hat{\omega}_\varphi-\frac{1}{2}\left(\frac{\partial\hat{\upsilon}_{\rm g}}{\partial z}-\frac{\partial\hat{u}_{\rm g}}{\partial r}\right)\right],$$

$$\hat{u}_{\rm p}\frac{\partial\hat{\omega}_r}{\partial z}+\hat{\upsilon}_{\rm p}\frac{\partial\hat{\omega}_r}{\partial r}-\frac{\hat{\omega}_\varphi\hat{w}_{\rm p}}{r}=-\gamma\left(\hat{\omega}_r-\frac{1}{2}\frac{\partial\hat{w}_{\rm g}}{\partial z}\right). \quad (8)$$

Applying the Reynolds procedure to the actual equations (8), subject to the assumptions made, we obtain the pulsational transfer equations of the moment of momentum of the solid phase along the radial and transversal axes:

$$u_{\rm p}\frac{\partial\omega'_\varphi}{\partial z}+u'_{\rm p}\frac{\partial\omega_\varphi}{\partial z}+u'_{\rm p}\frac{\partial\omega'_\varphi}{\partial z}+v_{\rm p}\frac{\partial\omega'_\varphi}{\partial r}+v'_{\rm p}\frac{\partial\omega_\varphi}{\partial r}+v'_{\rm p}\frac{\partial\omega'_\varphi}{\partial r}+\frac{\omega'_rw'_{\rm p}}{r}-\frac{\partial\langle\omega'_\varphi u'_{\rm p}\rangle}{\partial z}-\frac{\partial(r\langle\omega'_\varphi v'_{\rm p}\rangle)}{r\partial r}-$$

$$-\frac{\langle\omega'_rw'_{\rm p}\rangle}{r}=-\gamma\omega'_\varphi\,, \quad (9)$$

$$u_{\rm p}\frac{\partial\omega'_r}{\partial z}+u'_{\rm p}\frac{\partial\omega'_r}{\partial z}+v_{\rm p}\frac{\partial\omega'_r}{\partial r}+v'_{\rm p}\frac{\partial\omega'_r}{\partial r}-\frac{\omega_\varphi w'_{\rm p}}{r}-\frac{\omega'_\varphi w'_{\rm p}}{r}-\frac{\partial\langle\omega'_ru'_{\rm p}\rangle}{\partial z}-\frac{\partial(r\langle\omega'_rv'_{\rm p}\rangle)}{r\partial r}+\frac{\langle\omega'_\varphi w'_{\rm p}\rangle}{r}=-\gamma\omega'_r\,. \quad (10)$$

In order construct the transfer equation of the correlation $\langle\omega'_rw'_{\rm p}\rangle$ it is necessary to multiply Eq. (10) by $w'_{\rm p}$ and Eq. (6) by ω'_r and then sum up the resulting equations:

$$u_{\rm p}\frac{\partial\omega'_rw'_{\rm p}}{\partial z}+u'_{\rm p}\frac{\partial\omega'_rw'_{\rm p}}{\partial z}+v_{\rm p}\frac{\partial\omega'_rw'_{\rm p}}{\partial r}+v'_{\rm p}\frac{\partial\omega'_rw'_{\rm p}}{\partial r}-\frac{\omega_\varphi w'^2_{\rm p}}{r}-\frac{\omega'_\varphi w'^2_{\rm p}}{r}+\frac{w'_{\rm p}\langle\omega'_\varphi w'_{\rm p}\rangle}{r}+$$

$$+\frac{v_{\rm p}\omega'_rw'_{\rm p}}{r}+\frac{v'_{\rm p}\omega'_rw'_{\rm p}}{r}-\frac{w'_r\langle w'_{\rm p}v'_{\rm p}\rangle}{r}-w'_{\rm p}\frac{\partial\langle\omega'_ru'_{\rm p}\rangle}{\partial z}-w'_{\rm p}\frac{\partial(r\langle\omega'_rv'_{\rm p}\rangle)}{r\partial r}-\omega'_r\frac{\partial\langle u'_{\rm p}w'_{\rm p}\rangle}{\partial z}-$$

$$-\omega'_r\frac{\partial(r\langle w'_{\rm p}v'_{\rm p}\rangle)}{r\partial r}=\frac{F'_{\rm a\varphi}\omega'_r+F'_{\rm M\varphi}\omega'_r}{\rho_{\rm p}\beta}-\gamma\omega'_rw'_{\rm p}\,. \quad (11)$$

We will transform Eq. (11) with the aid of expression (7) and pulsational continuity equation preliminarily multiplied by $\omega'_rw'_{\rm p}$. Thereafter, in the resulting equations we take averages. Over the stretch of the stabilized motion of the two-phase flow we bring second moment of $\langle\omega'_rw'_{\rm p}\rangle$ to the form

$$\frac{\partial(r\langle\omega'_rw'_{\rm p}v'_{\rm p}\rangle)}{r\partial r}-\frac{\omega_\varphi\langle w'^2_{\rm p}\rangle}{r}-\frac{\langle\omega'_\varphi w'^2_{\rm p}\rangle}{r}+\frac{\langle\omega'_rw'_{\rm p}v'_{\rm p}\rangle}{r}=-\left(\frac{1}{\tau}+\gamma\right)\langle\omega'_rw'_{\rm p}\rangle+\lambda_\omega(u_{\rm g}-u_{\rm p})\langle\omega'^2_r\rangle\,. \quad (12)$$

Similarly we can obtain transfer equations for the remaining sought correlations: $\langle\omega'_rv'_{\rm p}\rangle$, $\langle\omega'_\varphi w'_{\rm p}\rangle$, $\langle\omega'_\varphi v'_{\rm p}\rangle$, $\langle\omega'^2_r\rangle$, $\langle\omega'^2_\varphi\rangle$, $\langle\omega'_\varphi\omega'_r\rangle$, $\langle w'^2_{\rm p}\rangle$, $\langle v'^2_{\rm p}\rangle$, and $\langle w'_{\rm p}v'_{\rm p}\rangle$. To close Eq. (12) it is necessary to compute triple correlations $\langle\omega'_rw'_{\rm p}v'_{\rm p}\rangle$ and $\langle\omega'_\varphi w'^2_{\rm p}\rangle$ present in that equation. For this purpose, we will construct transfer equations of the sought moments. We shall illustrate the derivation of these equations using as an example the equation for the variable $\langle\omega'_rw'_{\rm p}v'_{\rm p}\rangle$. We multiply the pulsational equation (10) by $w'_{\rm p}$ and Eq. (6) by ω'_r, and sum up the equations. We multiply the resulting expression by the pulsation of the radial velocity of particles $v'_{\rm p}$ and Eq. (5) by $\omega'_rw'_{\rm p}$ and sum up them. We will transform the constructed equation with the aid of Eq. (7) and pulsational continuity equation that had been preliminarily multiplied by $\omega'_rw'_{\rm p}v'_{\rm p}$. Thereafter, we perform averaging in the resulting equation. The transfer equation of the sought quantity $\langle\omega'_rw'_{\rm p}v'_{\rm p}\rangle$ for the stretch of the stabilized gas suspension flow is written as

$$\frac{\partial(r\langle\omega'_rw'_{\rm p}v'^2_{\rm p}\rangle)}{r\partial r}-\frac{\omega_\varphi\langle w'^2_{\rm p}v'_{\rm p}\rangle}{r}-\frac{\langle\omega'_\varphi w'^2_{\rm p}v'_{\rm p}\rangle}{r}+\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\langle\omega'_\varphi w'_{\rm p}\rangle}{r}+$$

$$+\frac{\langle \omega'_r w'_p v'^2_p \rangle}{r} - \frac{\langle \omega'_r v'_p \rangle \langle w'_p v'_p \rangle}{r} - \frac{\langle \omega'_r w'^3_p \rangle}{r} + \frac{\langle w'^2_p \rangle \langle \omega'_r w'_p \rangle}{r} - \frac{\langle w'_p v'_p \rangle \partial (r \langle \omega'_r v'_p \rangle)}{r\partial r} -$$

$$-\frac{\langle \omega'_r v'_p \rangle \partial (r \langle w'_p v'_p \rangle)}{r\partial r} - \frac{\langle \omega'_r w'_p \rangle \partial (r \langle v'^2_p \rangle)}{r\partial r} = -\psi_2 \langle \omega'_r w'_p v'_p \rangle +$$

$$+\lambda_\omega (u_g - u_p) (\langle \omega'^2_r v'_p \rangle - \langle \omega'_\varphi \omega'_r w'_p \rangle), \quad \psi_2 = \frac{1}{\tau} + \gamma . \tag{13}$$

In Eq. (13) there are fourth moments which, just as in [4], can be presented as a sum of the products of second moments:

$$\langle \omega'_r w'_p v'^2_p \rangle = \langle v'^2_p \rangle \langle \omega'_r w'_p \rangle + 2 \langle w'_p v'_p \rangle \langle \omega'_r v'_p \rangle ,$$

$$\langle \omega'_\varphi w'^2_p v'_p \rangle = 2 \langle w'_p v'_p \rangle \langle \omega'_\varphi w'_p \rangle + \langle w'^2_p \rangle \langle \omega'_\varphi v'_p \rangle , \tag{14}$$

$$\langle \omega'_r w'^3_p \rangle = 3 \langle w'^2_p \rangle \langle \omega'_r v'_p \rangle .$$

Substituting Eq. (14) into Eq. (13) and performing simple transformations, we obtain

$$\langle \omega'_r w'_p v'_p \rangle = -\frac{1}{\psi_2}\Bigg[\frac{\langle v'^2_p \rangle \partial \langle \omega'_r w'_p \rangle}{\partial r} + \frac{\langle \omega'_r v'_p \rangle \partial \langle w'_p v'_p \rangle}{\partial r} + \frac{\langle \omega'_r v'_p \rangle \langle w'_p v'_p \rangle}{r} +$$

$$+\frac{\langle w'_p v'_p \rangle \partial \langle \omega'_r v'_p \rangle}{\partial r} - \frac{\omega_\varphi \langle w'^2_p v'_p \rangle}{r} - \frac{\langle \omega'_\varphi v'_p \rangle \langle w'^2_p \rangle}{r} - \frac{\langle w'_p v'_p \rangle \langle \omega'_\varphi w'_p \rangle}{r} +$$

$$+\frac{\langle v'^2_p \rangle \langle \omega'_r w'_p \rangle}{r} - \frac{2 \langle w'^2_p \rangle \langle \omega'_r w'_p \rangle}{r} - \lambda_\omega (u_g - u_p) (\langle \omega'^2_r v'_p \rangle - \langle \omega'_\varphi \omega'_r w'_p \rangle)\Bigg]. \tag{15}$$

Similarly we can obtain algebraic expressions for the remaining unknown third moments present in the transfer equations of double correlations $\langle \omega'_r v'_p \rangle$, $\langle \omega'_r w'_p \rangle$, $\langle \omega'_\varphi w'_p \rangle$, $\langle \omega'_\varphi v'_p \rangle$, $\langle \omega'^2_r \rangle$,$\langle \omega'^2_\varphi \rangle$, $\langle \omega'_\varphi \omega'_r \rangle$, $\langle w'^2_p \rangle$, $\langle v'^2_p \rangle$, and $\langle w'_p v'_p \rangle$. Some of these equations are given below:

$$\langle \omega'_\varphi w'^2_p \rangle = -\frac{1}{\psi_1}\Bigg[\frac{\langle w'_p v'_p \rangle \partial \langle \omega'_\varphi w'_p \rangle}{\partial r} + \frac{\langle \omega'_\varphi v'_p \rangle \partial \langle w'^2_p \rangle}{2\partial r} + \frac{\langle w'^2_p v'_p \rangle \partial \omega_\varphi}{2\partial r} +$$

$$+\frac{\langle w'^2_p \rangle \langle \omega'_r w'_p \rangle}{r} + \frac{\langle w'_p v'_p \rangle \langle \omega'_\varphi w'_p \rangle}{r} + \frac{\langle w'^2_p \rangle \langle \omega'_\varphi v'_p \rangle}{r} - \lambda_\omega (u_g - u_p) \langle \omega'_\varphi \omega'_r w'_p \rangle\Bigg], \tag{16}$$

$$\langle \omega'_\varphi \omega'_r w'_p \rangle = -\frac{1}{\psi_3}\Bigg[\frac{\langle w'_p v'_p \rangle \partial \langle \omega'_\varphi \omega'_r \rangle}{\partial r} + \frac{\langle \omega'_\varphi v'_p \rangle \partial \langle \omega'_r w'_p \rangle}{\partial r} + \frac{\langle \omega'_r v'_p \rangle \partial \langle \omega'_\varphi w'_p \rangle}{\partial r} +$$

$$+\frac{\langle \omega'_r w'_p v'_p \rangle \partial \omega_\varphi}{\partial r} + \frac{\langle \omega'^2_r \rangle \langle w'^2_p \rangle}{r} + \frac{\langle \omega'_r w'_p \rangle^2}{r} - \frac{\omega_\varphi \langle \omega'_\varphi w'^2_p \rangle}{r} - \frac{\langle \omega'^2_\varphi \rangle \langle w'^2_p \rangle}{r} -$$

$$-\frac{\langle\omega'_\varphi w'_p\rangle^2}{r}+\frac{\langle\omega'_\varphi v'_p\rangle\,\langle\omega'_r w'_p\rangle}{r}+\frac{\langle\omega'_\varphi w'_p\rangle\,\langle\omega'_r v'_p\rangle}{r}-\lambda_\omega\,(u_g-u_p)\,\langle\omega'_\varphi\omega'^2_r\rangle\Bigg], \tag{17}$$

$$\langle\omega'^2_\varphi v'_p\rangle=-\frac{1}{\psi_4}\Bigg[\frac{\langle v'^2_p\rangle\,\partial\,\langle\omega'^2_\varphi\rangle}{2\partial r}+\frac{\langle\omega'_\varphi v'_p\rangle\,\partial\,\langle\omega'_\varphi v'_p\rangle}{\partial r}-\frac{\langle\omega'_\varphi w'_p\rangle^2}{r}+\frac{\langle\omega'_\varphi v'^2_p\rangle\,\partial\omega_\varphi}{\partial r}+$$

$$+\frac{\langle w'_p v'_p\rangle\,\langle\omega'_\varphi\omega'_r\rangle}{r}+\frac{\langle\omega'_r v'_p\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}-\frac{\lambda_\omega}{2}\,(u_g-u_p)\,\langle\omega'^3_\varphi\rangle\Bigg], \tag{18}$$

$$\langle\omega'^2_r w'_p\rangle=-\frac{1}{\psi_4}\Bigg[\frac{\langle\omega'_r v'_p\rangle\,\partial\,\langle\omega'_r w'_p\rangle}{\partial r}+\frac{\langle w'_p v'_p\rangle\,\partial\,\langle\omega'^2_r\rangle}{2\partial r}-\frac{\omega_\varphi\,\langle\omega'_r w'^2_p\rangle}{r}-$$

$$-\frac{\langle\omega'_\varphi\omega'_r\rangle\,\langle w'^2_p\rangle}{r}-\frac{\langle\omega'_\varphi w'_p\rangle\,\langle\omega'_r w'_p\rangle}{r}+\frac{\langle\omega'_r v'_p\rangle\,\langle\omega'_r w'_p\rangle}{r}-\frac{\lambda_\omega}{2}\,(u_g-u_p)\,\langle\omega'^3_r\rangle\Bigg], \tag{19}$$

$$\langle\omega'^2_\varphi w'_p\rangle=-\frac{1}{\psi_4}\Bigg[\frac{\langle w'_p v'_p\rangle\,\partial\,\langle\omega'^2_\varphi\rangle}{2\partial r}+\frac{\langle\omega'_\varphi v'_p\rangle\,\partial\,\langle\omega'_\varphi w'_p\rangle}{\partial r}+\frac{\langle\omega'_\varphi w'_p v'_p\rangle\,\partial\omega_\varphi}{\partial r}+$$

$$+\frac{\langle\omega'_\varphi\omega'_r\rangle\,\langle w'^2_p\rangle}{r}+\frac{\langle\omega'_r w'_p\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}+\frac{\langle\omega'_\varphi v'_p\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}-\frac{\lambda_\omega}{2}\,(u_g-u_p)\,\langle\omega'^2_\varphi\omega'_r\rangle\Bigg], \tag{20}$$

$$\langle\omega'_r w'^2_p\rangle=-\frac{1}{\psi_1}\Bigg[\frac{\langle\omega'_r v'_p\rangle\,\partial\,\langle w'^2_p\rangle}{2\partial r}+\frac{\langle w'_p v'_p\rangle\,\partial\,\langle\omega'_r w'_p\rangle}{\partial r}+\frac{\langle\omega'_r v'_p\rangle\,\langle w'^2_p\rangle}{r}+$$

$$+\frac{\langle w'_p v'_p\rangle\,\langle\omega'_r w'_p\rangle}{r}-\frac{\omega_\varphi\,\langle w'^3_p\rangle}{2r}-\frac{\langle w'^2_p\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}-\lambda_\omega\,(u_g-u_p)\,\langle\omega'^2_r w'_p\rangle\Bigg], \tag{21}$$

$$\langle\omega'_\varphi\omega'^2_r\rangle=-\frac{1}{\psi_6}\Bigg[\frac{\langle\omega'_r v'_p\rangle\,\partial\,\langle\omega'_\varphi\omega'_r\rangle}{\partial r}+\frac{\langle\omega'_\varphi v'_p\rangle\,\partial\,\langle\omega'^2_r\rangle}{2\partial r}-\frac{\omega_\varphi\,\langle\omega'_\varphi\omega'_r w'_p\rangle}{r}-$$

$$-\frac{\langle\omega'^2_\varphi\rangle\,\langle\omega'_r w'_p\rangle}{r}-\frac{\langle\omega'_\varphi\omega'_r\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}+\frac{\langle\omega'^2_r v'_p\rangle\,\partial\omega_\varphi}{2\partial r}+\frac{\langle\omega'^2_r\rangle\,\langle\omega'_r w'_p\rangle}{r}\Bigg], \tag{22}$$

$$\langle\omega'^2_\varphi\omega'_r\rangle=-\frac{1}{\psi_6}\Bigg[\frac{\langle\omega'_\varphi v'_p\rangle\,\partial\,\langle\omega'_\varphi\omega'_r\rangle}{\partial r}+\frac{\langle\omega'_r v'_p\rangle\,\partial\,\langle\omega'^2_\varphi\rangle}{2\partial r}+\frac{\langle\omega'_\varphi\omega'_r v'_p\rangle\,\partial\omega_\varphi}{\partial r}+$$

$$+\frac{\langle\omega'_\varphi\omega'_r\rangle\,\langle\omega'_r w'_p\rangle}{r}+\frac{\langle\omega'^2_r\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}-\frac{\omega_\varphi\,\langle\omega'^2_\varphi w'_p\rangle}{2r}-\frac{\langle\omega'^2_\varphi\rangle\,\langle\omega'_\varphi w'_p\rangle}{r}\Bigg], \tag{23}$$

$$\langle \omega_r'^3 \rangle = -\frac{1}{\gamma}\left[\frac{\langle \omega_r' v_p' \rangle \, \partial \, \langle \omega_r'^2 \rangle}{\partial r} - \frac{\omega_\varphi \, \langle \omega_r'^2 w_p' \rangle}{r} - \frac{2 \, \langle \omega_\varphi' \omega_r' \rangle \, \langle \omega_r' w_p' \rangle}{r}\right], \tag{24}$$

$$\langle \omega_\varphi'^3 \rangle = -\frac{1}{\gamma}\left[\frac{\langle \omega_\varphi' v_p' \rangle \, \partial \, \langle \omega_\varphi'^2 \rangle}{\partial r} + \frac{\langle \omega_\varphi'^2 v_p' \rangle \, \partial \omega_\varphi}{\partial r} + \frac{2 \, \langle \omega_\varphi' \omega_r' \rangle \, \langle \omega_\varphi' w_p' \rangle}{r}\right], \tag{25}$$

where

$$\psi_1 = \frac{1}{\tau} + \frac{\gamma}{2}\,; \quad \psi_3 = \frac{1}{\tau} + 2\gamma\,; \quad \psi_4 = \frac{1}{2\tau} + \gamma\,; \quad \psi_6 = \frac{3\gamma}{2}\,.$$

We transform Eq. (12) with the aid of Eq. (15). As a result, we arrive at the parabolic transfer equation of the correlation $\langle \omega_r' w_p' \rangle$:

$$-\frac{\partial}{r\partial r}\left(\frac{r \, \langle v_p'^2 \rangle \, \partial \, \langle \omega_r' w_p' \rangle}{\psi_2 \partial r}\right) - \frac{\partial}{r\partial r}\left(\frac{r \, \langle \omega_r' v_p' \rangle \, \partial \, \langle w_p' v_p' \rangle}{\psi_2 \partial r}\right) - \frac{\partial}{r\partial r}\frac{\langle \omega_r' v_p' \rangle \, \langle w_p' v_p' \rangle}{\psi_2} -$$

$$-\frac{\partial}{r\partial r}\left(\frac{r \, \langle w_p' v_p' \rangle \, \partial \, \langle \omega_r' v_p' \rangle}{\psi_2 \partial r}\right) + \frac{\partial}{r\partial r}\frac{\omega_\varphi \, \langle w_p'^2 v_p' \rangle}{\psi_2} + \frac{\partial}{r\partial r}\frac{\langle \omega_\varphi' v_p' \rangle \, \langle w_p'^2 \rangle}{\psi_2} +$$

$$+\frac{\partial}{r\partial r}\frac{\langle w_p' v_p' \rangle \, \langle \omega_\varphi' w_p' \rangle}{\psi_2} - \frac{\partial}{r\partial r}\frac{\langle v_p'^2 \rangle \, \langle \omega_r' w_p' \rangle}{\psi_2} + \frac{2\partial}{r\partial r}\frac{\langle w_p'^2 \rangle \, \langle \omega_r' w_p' \rangle}{\psi_2} +$$

$$+\frac{\lambda_\omega \partial r \, (u_g - u_p) \, (\langle w_r'^2 v_p' \rangle - \langle \omega_\varphi' \omega_r' w_p' \rangle)}{r \partial r \psi_2} - \frac{\omega_\varphi \, \langle w_p'^2 \rangle}{r} - \frac{\langle \omega_\varphi' w_p'^2 \rangle}{r} + \frac{\langle \omega_r' w_p' v_p' \rangle}{r} =$$

$$= -\left(\frac{1}{\tau} + \gamma\right)\langle \omega_r' w_p' \rangle + \lambda_\omega \, (u_g - u_p) \, \langle \omega_r'^2 \rangle \,. \tag{26}$$

Similarly we can obtain parabolic transfer equations for the remaining sought correlations: $\langle \omega_\varphi' w_p' \rangle$, $\langle \omega_r'^2 \rangle$, $\langle \omega_\varphi'^2 \rangle$, $\langle \omega_\varphi' v_p' \rangle$, $\langle \omega_\varphi' \omega_r' \rangle$, $\langle \omega_r' v_p' \rangle$, $\langle v_p'^2 \rangle$, $\langle w_p'^2 \rangle$, and $\langle w_p' v_p' \rangle$. Below they are given without derivation:

the quantity $\langle \omega_\varphi' v_p' \rangle$

$$-\frac{\partial}{r\partial r}\left(\frac{r \, \langle v_p'^2 \rangle \, \partial \, \langle \omega_\varphi' w_p' \rangle}{\psi_5 \partial r}\right) - \frac{\partial}{r\partial r}\left(\frac{r \, \langle \omega_\varphi' v_p' \rangle \, \partial \, \langle w_p' v_p' \rangle}{\psi_5 \partial r}\right) - \frac{\partial}{r\partial r}\left(\frac{\langle \omega_\varphi' v_p' \rangle \, \langle w_p' v_p' \rangle}{\psi_5}\right) -$$

$$-\frac{\partial}{r\partial r}\left(\frac{r \, \langle w_p' v_p' \rangle \, \partial \, \langle \omega_\varphi' v_p' \rangle}{\psi_5 \partial r}\right) + \frac{2\partial}{r\partial r}\frac{\langle w_p'^2 \rangle \, \langle \omega_\varphi' w_p' \rangle}{\psi_5} - \frac{\partial}{r\partial r}\frac{\langle v_p'^2 \rangle \, \langle \omega_\varphi' w_p' \rangle}{\psi_5} -$$

$$-\frac{\partial}{r\partial r}\left(\frac{r \, \langle v_p'^2 w_p' \rangle \, \partial \omega_\varphi}{\psi_5 \partial r}\right) - \frac{\partial}{r\partial r}\frac{\langle w_p' v_p' \rangle \, \langle \omega_r' w_p' \rangle}{\psi_5} - \frac{\partial}{r\partial r}\frac{\langle w_p'^2 \rangle \, \langle \omega_r' v_p' \rangle}{\psi_5} +$$

$$+\frac{\lambda_\omega \partial r \, (u_g - u_p) \, (\langle \omega_\varphi' \omega_r' v_p' \rangle - \langle \omega_\varphi'^2 w_p' \rangle)}{r \partial r \psi_5} + \frac{\langle w_p' v_p' \rangle \, \partial \omega_\varphi}{\partial r} + \frac{\langle \omega_r' w_p'^2 \rangle}{r} + \frac{\langle \omega_\varphi' w_p' v_p' \rangle}{r} =$$

$$= -\left(\frac{1}{\tau} + \gamma\right) \langle \omega'_\varphi w'_\text{p} \rangle + \lambda_\omega (u_\text{g} - u_\text{p}) \langle \omega'_\varphi \omega'_r \rangle , \quad \psi_5 = \frac{2}{\tau} + \gamma ; \tag{27}$$

the quantity $\langle \omega'^2_r \rangle$

$$-\frac{\partial}{4r\partial r}\left(\frac{r \langle v'^2_\text{p} \rangle \partial \langle \omega'^2_r \rangle}{\psi_4 \partial r}\right) - \frac{\partial}{2r\partial r}\left(\frac{r \langle \omega'_r v'_\text{p} \rangle \partial \langle \omega'_r v'_\text{p} \rangle}{\psi_4 \partial r}\right) + \frac{\partial}{2r\partial r}\frac{\omega_\varphi \langle \omega'_r w'_\text{p} v'_\text{p} \rangle}{\psi_4} +$$

$$+ \frac{\partial}{2r\partial r}\frac{\langle w'_\text{p} v'_\text{p} \rangle \langle \omega'_\varphi \omega'_r \rangle}{\psi_4} + \frac{\partial}{2r\partial r}\frac{\langle \omega'_\varphi v'_\text{p} \rangle \langle \omega'_r w'_\text{p} \rangle}{\psi_4} + \frac{\partial}{2r\partial r}\frac{\langle \omega'_r w'_\text{p} \rangle^2}{\psi_4} -$$

$$- \frac{\lambda_\omega \partial r (u_\text{g} - u_\text{p}) \langle \omega'_\varphi \omega'^2_r \rangle}{4r\partial r \psi_4} - \frac{\omega_\varphi \langle \omega'_r w'_\text{p} \rangle}{r} - \frac{\langle \omega'_\varphi \omega'_r \omega'_\text{p} \rangle}{r} = -\gamma \langle \omega'^2_r \rangle ; \tag{28}$$

the quantity $\langle \omega'^2_\varphi \rangle$

$$-\frac{\partial}{4r\partial r}\left(\frac{r \langle v'^2_\text{p} \rangle \partial \langle \omega'^2_\varphi \rangle}{\psi_4 \partial r}\right) - \frac{\partial}{2r\partial r}\left(\frac{r \langle \omega'_\varphi v'_\text{p} \rangle \partial \langle \omega'_\varphi v'_\text{p} \rangle}{\psi_4 \partial r}\right) + \frac{\partial}{2r\partial r}\frac{\langle \omega'_\varphi w'_\text{p} \rangle^2}{\psi_4} -$$

$$- \frac{\partial}{2r\partial r}\left(\frac{r \langle \omega'_\varphi v'^2_\text{p} \rangle \partial \omega_\varphi}{\psi_4 \partial r}\right) - \frac{\partial}{2r\partial r}\frac{\langle w'_\text{p} v'_\text{p} \rangle \langle \omega'_\varphi \omega'_r \rangle}{\psi_4} - \frac{\partial}{2r\partial r}\frac{\langle \omega'_r v'_\text{p} \rangle \langle \omega'_\varphi w'_\text{p} \rangle}{\psi_4} +$$

$$+ \frac{\lambda_\omega \partial r (u_\text{g} - u_\text{p}) \langle \omega'^3_\varphi \rangle}{4r\partial r \psi_4} + \frac{\langle \omega'_\varphi v'_\text{p} \rangle \partial \omega_\varphi}{\partial r} + \frac{\langle \omega'_\varphi \omega'_r w'_\text{p} \rangle}{r} = -\gamma \langle \omega'^2_\varphi \rangle ; \tag{29}$$

the quantity $\langle \omega'_\varphi v'_\text{p} \rangle$

$$-\frac{\partial}{r\partial r}\left(\frac{r \langle v'^2_\text{p} \rangle \partial \langle \omega'_\varphi v'_\text{p} \rangle}{\psi_1 \partial r}\right) - \frac{\partial}{2r\partial r}\left(\frac{r \langle \omega'_\varphi v'_\text{p} \rangle \partial \langle v'^2_\text{p} \rangle}{\psi_1 \partial r}\right) - \frac{\partial}{2r\partial r}\left(\frac{r \langle v'^3_\text{p} \rangle \partial \omega_\varphi}{\psi_1 \partial r}\right) -$$

$$- \frac{\partial}{r\partial r}\frac{\langle w'_\text{p} v'_\text{p} \rangle \langle \omega'_r v'_\text{p} \rangle}{\psi_1} + \frac{2\partial}{r\partial r}\frac{\langle w'_\text{p} v'_\text{p} \rangle \langle \omega'_\varphi w'_\text{p} \rangle}{\psi_1} - \frac{\lambda_\omega \partial r (u_\text{g} - u_\text{p}) \langle \omega'^2_\varphi v'_\text{p} \rangle}{r\partial r \psi_1} +$$

$$+ \frac{\langle v'^2_\text{p} \rangle \partial \omega_\varphi}{\partial r} - \frac{\langle \omega'_\varphi w'^2_\text{p} \rangle}{r} + \frac{\langle \omega'_r w'_\text{p} v'_\text{p} \rangle}{r} = -\left(\frac{1}{\tau} + \gamma\right) \langle \omega'_\varphi v'_\text{p} \rangle - \lambda_\omega (u_\text{g} - u_\text{p}) \langle \omega'^2_\varphi \rangle ; \tag{30}$$

the quantity $\langle \omega'_\varphi \omega'_r \rangle$

$$-\frac{\partial}{r\partial r}\left(\frac{r \langle v'^2_\text{p} \rangle \partial \langle \omega'_\varphi \omega'_r \rangle}{\psi_3 \partial r}\right) - \frac{\partial}{r\partial r}\left(\frac{r \langle \omega'_\varphi v'_\text{p} \rangle \partial \langle \omega'_r v'_\text{p} \rangle}{\psi_3 \partial r}\right) - \frac{\partial}{r\partial r}\left(\frac{r \langle \omega'_r v'_\text{p} \rangle \partial \langle \omega'_\varphi v'_\text{p} \rangle}{\psi_3 \partial r}\right) +$$

$$+ \frac{2\partial}{r\partial r}\frac{\langle \omega'_\varphi w'_\text{p} \rangle \langle \omega'_r w'_\text{p} \rangle}{\psi_3} - \frac{\partial}{r\partial r}\left(\frac{r \langle \omega'_r v'^2_\text{p} \rangle \partial \omega_\varphi}{\psi_3 \partial r}\right) - \frac{\partial}{r\partial r}\frac{\langle \omega'^2_r \rangle \langle w'_\text{p} v'_\text{p} \rangle}{\psi_3} -$$

$$
-\frac{\partial}{r\partial r}\frac{\langle\omega'_r w'_p\rangle\,\langle\omega'_r v'_p\rangle}{\psi_3}+\frac{\partial}{r\partial r}\frac{\omega_\varphi\,\langle\omega'_\varphi w'_p v'_p\rangle}{\psi_3}+\frac{\partial}{r\partial r}\frac{\langle\omega'^2_\varphi\rangle\,\langle w'_p v'_p\rangle}{\psi_3}+
$$

$$
+\frac{\partial}{r\partial r}\frac{\langle\omega'_\varphi w'_p\rangle\,\langle\omega'_\varphi v'_p\rangle}{\psi_3}-\frac{\lambda_\omega\partial r\,(u_g-u_p)\,\langle\omega'^2_\varphi\omega'_r\rangle}{r\partial r\psi_3}+\frac{\langle\omega'_r v'_p\rangle\,\partial\omega_\varphi}{\partial r}+\frac{\langle\omega'^2_r w'_p\rangle}{r}-
$$

$$
-\frac{\omega_\varphi\,\langle\omega'_\varphi w'_p\rangle}{r}-\frac{\langle\omega'^2_\varphi w'_p\rangle}{r}=-2\gamma\,\langle\omega'_\varphi\omega'_r\rangle\,; \tag{31}
$$

the quantity $\langle\omega'_r v'_p\rangle$

$$
-\frac{\partial}{r\partial r}\left(\frac{r\,\langle v'^2_p\rangle\,\partial\,\langle\omega'_r v'_p\rangle}{\psi_1\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\,\langle\omega'_r v'_p\rangle\,\partial\,\langle v'^2_p\rangle}{\psi_1\partial r}\right)+\frac{2\partial}{r\partial r}\frac{\langle\omega'_r w'_p\rangle\,\langle w'_p v'_p\rangle}{\psi_1}+
$$

$$
+\frac{\partial}{2r\partial r}\frac{\omega_\varphi\,\langle v'^2_p w'_p\rangle}{\psi_1}+\frac{\partial}{r\partial r}\frac{\langle w'_p v'_p\rangle\,\langle\omega'_\varphi v'_p\rangle}{\psi_1}-\frac{\lambda_\omega\partial r\,(u_g-u_p)\,\langle\omega'_\varphi\omega'_r v'_p\rangle}{r\partial r\psi_1}-
$$

$$
-\frac{\omega_\varphi\,\langle w'_p v'_p\rangle}{r}-\frac{\langle\omega'_\varphi w'_p v'_p\rangle}{r}-\frac{\langle\omega'_r w'^2_p\rangle}{r}=-\left(\frac{1}{\tau}+\gamma\right)\langle\omega'_r v'_p\rangle-\lambda_\omega\,(u_g-u_p)\,\langle\omega'_\varphi\omega'_r\rangle\,; \tag{32}
$$

the quantity $\langle v'_p v'_p\rangle$

$$
\frac{\partial}{r\partial r}\left(r\tau\,\langle v'_p v'_p\rangle\,\frac{\partial\,\langle v'_p v'_p\rangle}{\partial r}\right)-\frac{2}{r}\frac{\partial\,(\tau\,\langle w'_p v'_p\rangle^2)}{\partial r}-\frac{2\tau\,\langle v'_p v'_p\rangle}{3r}\frac{\partial\,\langle w'_p w'_p\rangle}{\partial r}-
$$

$$
-\frac{4\tau\,\langle w'_p v'_p\rangle}{3r}\frac{\partial\,\langle w'_p v'_p\rangle}{\partial r}+\frac{4\tau\,\langle w'_p w'_p\rangle^2}{3r^2}-\frac{4\tau\,\langle v'_p v'_p\rangle\,\langle w'_p w'_p\rangle}{3r^2}-\frac{4\tau\,\langle w'_p v'_p\rangle^2}{3r^2}+\frac{2}{\tau}\,(\langle v'_p v'_g\rangle-\langle v'_p v'_p\rangle)-
$$

$$
-2\lambda_\omega\,(u_g-u_p)\,\langle\omega'_\varphi v'_p\rangle+2N\left\{\frac{\delta^2}{6912\beta^2}\left(\frac{\partial u_p}{\partial r}\right)^2\left(\frac{1-K_n}{2}-\frac{1-K_\tau}{7}\right)^2-C_1\,\langle v'^2_p\rangle\,(1-K_n^2)\right\}+
$$

$$
+\frac{\lambda_\omega\partial r\tau\,\langle\omega'_\varphi v'^2_p\rangle\,(u_g-u_p)}{r\partial r}+\frac{4\tau\lambda_\omega\,(u_g-u_p)\,(\langle\omega'_r w'_p v'_p\rangle-\langle\omega'_\varphi w'^2_p\rangle)}{3r}=0\,; \tag{33}
$$

the quantity $\langle w'^2_p\rangle$

$$
\frac{\partial}{3r\partial r}\left(r\tau\,\langle v'_p v'_p\rangle\,\frac{\partial\,\langle w'^2_p\rangle}{\partial r}\right)+\frac{2\partial}{3r\partial r}\left(r\tau\,\langle w'_p v'_p\rangle\,\frac{\partial\,\langle w'_p v'_p\rangle}{\partial r}\right)-\frac{2}{3r}\frac{\partial\,(\tau\,\langle w'_p w'_p\rangle^2)}{\partial r}+
$$

$$
+\frac{2}{3r}\frac{\partial\,(\tau\,\langle v'_p v'_p\rangle\,\langle w'_p w'_p\rangle)}{\partial r}+\frac{2}{3r}\frac{\partial\,(\tau\,\langle w'_p v'_p\rangle^2)}{\partial r}+\frac{2\tau\,\langle v'_p v'_p\rangle}{3r}\frac{\partial\,\langle w'_p w'_p\rangle}{\partial r}+
$$

$$
+\frac{4\tau\,\langle w'_p v'_p\rangle}{3r}\frac{\partial\,\langle w'_p v'_p\rangle}{\partial r}-\frac{4\tau\,\langle w'_p w'_p\rangle^2}{3r^2}+\frac{4\tau\,\langle v'_p v'_p\rangle\,\langle w'_p w'_p\rangle}{3r^2}+\frac{4\tau\,\langle w'_p v'_p\rangle^2}{3r^2}+\frac{2}{\tau}\times
$$

$$\times(\langle w'_p w'_g\rangle - \langle w'_p w'_p\rangle) + 2\lambda_\omega (u_g - u_p) \langle \omega'_r w'_p\rangle +$$

$$+ 2N\left\{\frac{\delta^2}{6912\beta^2}\left(\frac{\partial u_p}{\partial r}\right)^2\left(\frac{1-K_n}{2} - \frac{1-K_\tau}{7}\right)^2 - C_2 \langle w'^2_p\rangle (1-K_n^2)\right\} -$$

$$- \frac{2\lambda_\omega \partial r\tau (u_g - u_p)(\langle \omega'_r w'_p v'_p\rangle - \langle \omega'_\varphi w'^2_p\rangle)}{3r\partial r} - \frac{4\tau\lambda_\omega (u_g - u_p)(\langle \omega'_r w'_p v'_p\rangle - \langle \omega'_\varphi w'^2_p\rangle)}{3r} = 0\ ; \quad (34)$$

the quantity $\langle w'_p v'_p\rangle$

$$\frac{2\partial}{3r\partial r}\left(r\tau \langle v'_p v'_p\rangle \frac{\partial \langle w'_p v'_p\rangle}{\partial r}\right) + \frac{2}{3r}\frac{\partial(\tau\langle v'_p v'_p\rangle \langle w'_p v'_p\rangle)}{\partial r} + \frac{\partial}{3r\partial r}\left(r\tau\langle w'_p v'_p\rangle \frac{\partial \langle v'_p v'_p\rangle}{\partial r}\right) -$$

$$- \frac{4}{3r}\frac{\partial(\tau\langle w'_p v'_p\rangle \langle w'^2_p\rangle)}{\partial r} - \frac{\tau\langle w'_p v'_p\rangle}{r}\frac{\partial\langle w'_p w'_p\rangle}{\partial r} - \frac{10\tau\langle w'_p v'_p\rangle\langle w'_p w'_p\rangle}{3r^2} +$$

$$+ \frac{2\tau\langle v'_p v'_p\rangle}{3r}\frac{\partial\langle w'_p v'_p\rangle}{\partial r} + \frac{2\tau\langle v'_p v'_p\rangle\langle w'_p v'_p\rangle}{3r^2} + \lambda_\omega (u_g - u_p)(\langle \omega'_r v'_p\rangle - \langle \omega'_\varphi w'_p\rangle) +$$

$$+ \frac{\tau\langle w'_p v'_p\rangle}{3r}\frac{\partial\langle v'_p v'_p\rangle}{\partial r} + \frac{\langle v'_g w'_p\rangle + \langle v'_p w'_g\rangle - 2\langle w'_p v'_p\rangle}{\tau} + \frac{\lambda_\omega \tau \langle \omega'_r w'^2_p\rangle (u_g - u_p)}{r} -$$

$$- \frac{2\lambda_\omega \partial r\tau (u_g - u_p)}{3r\partial r}\left(\frac{\langle \omega'_r v'^2_p\rangle}{2} - \langle \omega'_\varphi w'_p v'_p\rangle\right) - \frac{2\lambda_\omega \tau (u_g - u_p)}{3r}\left(\frac{\langle \omega'_r v'^2_p\rangle}{2} - \langle \omega'_\varphi w'_p v'_p\rangle\right) = 0\ . \quad (35)$$

The mixed correlation moments $\langle v'_g w'_p\rangle$, $\langle v'_p w'_g\rangle$, $\langle w'_g w'_p\rangle$, and $\langle v'_g v'_p\rangle$ are determined according to the recommendations given in [6]. In Eqs. (33) and (34) there are terms that describe the generation and dissipation of the pseudoturbulent energy of the solid phase caused by interparticle collisions due to their averaged and pulsating motion (the eleventh and twelfth terms of Eq. (33) and the fourteenth and fifteenth terms of Eq. (34)). These terms cannot be calculated by the methods of turbulence theory, since the pulsations which are associated with interparticle collisions depend mainly on the random position of the unit vector directed along the impact line. Therefore, to determine them a specially developed computational technique was used, which is based on the analysis of the dynamics of the process of collisions [8].

In order to calculate a two-phase flow over a stabilized portion of a tube with the aid of Eqs. (1)–(4), (15)–(35), we must specify boundary conditions on the axis and channel wall. As the boundary conditions on the tube axis the requirements for the flow symmetry are used:

$$(\partial u_g/\partial r)_{ax} = (\partial k_g/\partial r)_{ax} = (\partial \langle v'^2_p\rangle/\partial r)_{ax} = (\partial\langle w'^2_p\rangle/\partial r)_{ax} = (\partial\langle w'_p v'_p\rangle/\partial r)_{ax} = 0\ ,$$

$$(\partial\langle \omega'_\varphi v'_p\rangle/\partial r)_{ax} = (\partial\langle \omega'_\varphi \omega'_r\rangle/\partial r)_{ax} = (\partial\langle \omega'_r v'_p\rangle/\partial r)_{ax} = (\partial u_p/\partial r)_{ax} = (\partial \varepsilon_g/\partial r)_{ax} = 0\ , \quad (36)$$

$$(\partial\langle \omega'_\varphi w'_p\rangle/\partial r)_{ax} = (\partial\langle \omega'_r w'_p\rangle/\partial r)_{ax} = (\partial\langle w'^2_r\rangle/\partial r)_{ax} = (\partial\langle \omega'^2_\varphi\rangle/\partial r)_{ax} = 0\ ,$$

and on the channel wall the following relationships are given:

$$\varepsilon_{g,w} = \eta_g\ (\partial^2 k_g/\partial r^2)_w\ ,\quad u_{g,w} = k_{g,w} = 0\ ,\quad u_{p,w} = \frac{\delta}{24\sqrt{2}\ \beta(1-K_\tau)}\left(\frac{\partial u_p}{\partial r}\right)_w (7K_n - 2K_\tau - 5)\ ,$$

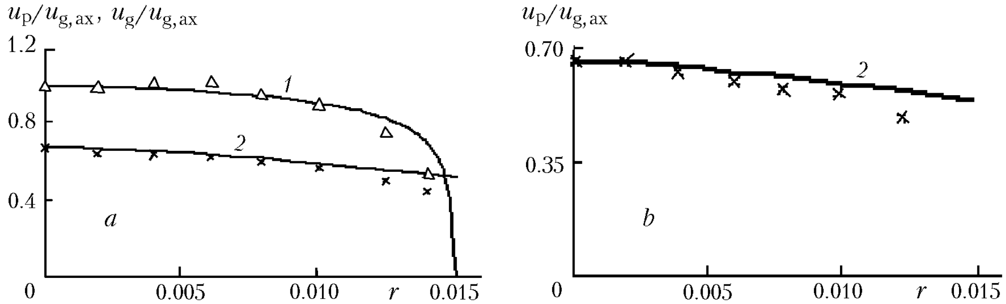
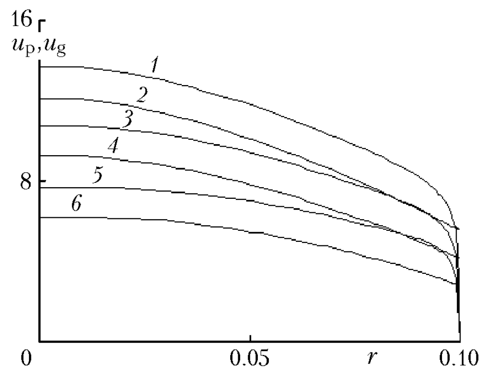


Fig. 1. Distribution of the axial velocities of the carrying medium 1 and dispersed phase 2 over the cross section of flow in comparison with the experimental data of [9] at the mass concentration of particles: a) 1.1; b) 2 (points, experiment).



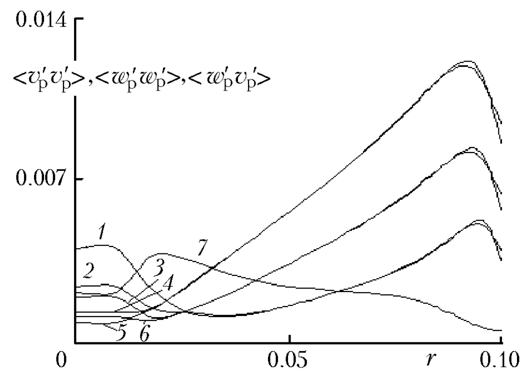


Fig. 2. Profiles of the longitudinal velocities of the gas and particles: variant I — 5) u_g; 6) u_p; II — 1) u_g; 2) u_p; III — 3) u_g; 4) u_p.

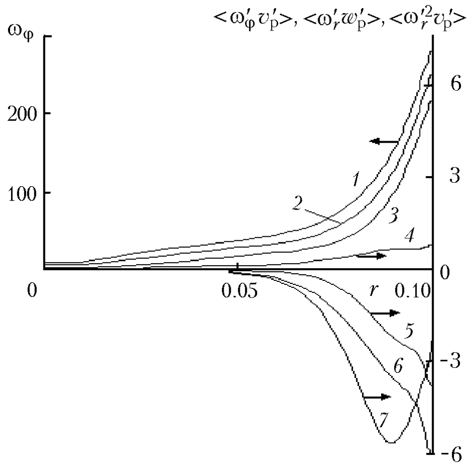
Fig. 3. Profiles of the second moments of pulsations of the translational velocity of the solid phase: variant I — 1) $\langle w'_p w'_p \rangle$; 2) $\langle v'_p v'_p \rangle$; 7) $\langle w'_p v'_p \rangle$; II — 4) $\langle w'_p w'_p \rangle$; 5) $\langle v'_p v'_p \rangle$; III — 3) $\langle w'_p w'_p \rangle$; 6) $\langle v'_p v'_p \rangle$.

$$(\partial \langle v'^2_p \rangle / \partial r)_w = (\partial \langle w'^2_p \rangle / \partial r)_w = (\partial \langle w'_p v'_p \rangle / \partial r)_w = (\partial \langle \omega'_\varphi v'_p \rangle / \partial r)_w = 0 \,,$$

$$(\partial \langle \omega'_\varphi \omega'_r \rangle / \partial r)_w = (\partial \langle \omega'_r v'_p \rangle / \partial r)_w = (\partial \langle \omega'_\varphi w'_p \rangle / \partial r)_w = (\partial \langle \omega'_r w'_p \rangle / \partial r)_w = 0 \,, \quad (37)$$

$$(\partial \langle \omega'^2_r \rangle / \partial r)_w = (\partial \langle \omega'^2_\varphi \rangle / \partial r)_w = 0 \,.$$

The foregoing system of equations was integrated numerically by the pivot method with iterations on a nonuniform grid with 51 nodes over the coordinate r. The nonuniformity of the grid was prescribed so that not less than 5 points could be present in the region of the viscous sublayer. Let us discuss the results of calculations of three variants at $\beta = 0.0012$, $\rho_g = 1.3$ kg/m^3 and $\rho_p = 1400$ kg/m^3, $K_\tau = 0.3$, $K_n = 0.5$, $R = 0.1$ m, and $\delta = 0.29 \cdot 10^{-3}$ m. Variant I) $u_{g,m} = 6$ m/sec; II) $u_{g,m} = 10$ m/sec; III) $u_{g,m} = 8$ m/sec. The calculation procedure is illustrated in Figs. 1–7 in which the profiles of the averaged and pulsating characteristics of a two-phase flow are shown. Figure 1 presents the calculated values of the averaged velocities of the gas and particles in comparison with the data of measurements of [9] for a vertical channel with $R = 15$ mm at $u_{g,m} = 8$ m/sec, $\delta = 0.5 \cdot 10^{-3}$ m, and $\rho_p = 1020$ kg/m^3. It is seen that the model gives a good description of the qualitative and quantitative behavior of the curves. Some discrepancies between the results of calculations and experimental data in the wall zone seem to be associated with the fact that in the model the assumption on uniform distribution of the concentration of the solid phase over the channel section is made.



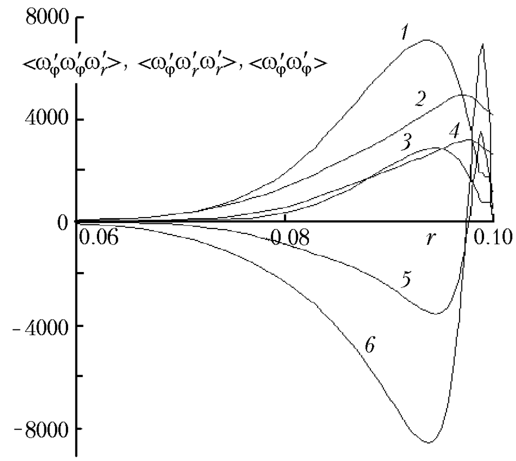


Fig. 4. Profiles of the averaged angular velocity of particles and of mixed correlations of second and third order: variant I — 3) ω_φ; 5) $\langle\omega'_\varphi v'_p\rangle$; II — 1) ω_φ; III — 2) ω_φ; 4) $\langle\omega'_r w'_p\rangle$; 6) $\langle\omega'_\varphi v'_p\rangle$; 7) $\langle\omega'^2_r v'_p\rangle$.

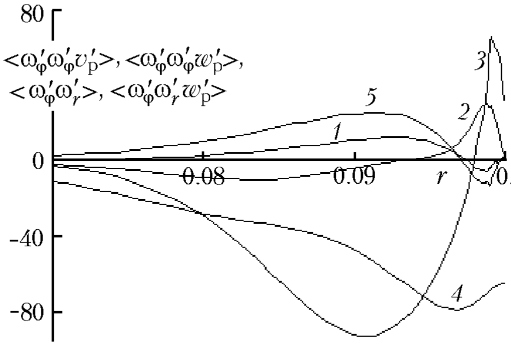
Fig. 5. Profiles of pulsating moments of the rotational velocity of particles: variant I — 3) $\langle\omega'_\varphi\omega'^2_r\rangle$; 4) $\langle\omega'^2_\varphi\rangle$; 5) $\langle\omega'^2_\varphi\omega'_r\rangle$; III — 1) $\langle\omega'_\varphi\omega'^2_r\rangle$; 2) $\langle\omega'^2_\varphi\rangle$; 6) $\langle\omega'^2_\varphi\omega'_r\rangle$.

The profiles of the longitudinal velocities of the gaseous and dispersed phases over the stretch of the stabilized gas suspension flow are shown in Fig. 2. Near the channel wall, where the gas velocity tends to zero and the carrying medium is unable to transport particles, they are suspended due to the pulsating transfer of the solid phase momentum. On the whole, the character of change in the function $u_g(r)$ is close to the character of change in the $u_p(r)$ curves. This similarity is preserved with an increase in the gas velocity.

Figure 3 presents the values of the shear, $\langle w'_p v'_p\rangle$, and normal, $\langle w'_p w'_p\rangle$, $\langle v'_p v'_p\rangle$ Reynolds stresses over the the stabilized stretch of the channel. It is seen that in the axial zone the field of the pulsation energy of the solid phase is anisotropic, and in the peripheral region it is isotropic. The presence of distinct maxima of the $\langle v'_p v'_p\rangle(r)$ and $\langle w'_p w'_p\rangle(r)$ curves in the wall zone is due to the influence of generation of the pseudoturbulent energy of particles due to interparticle collisions $\dfrac{2N\delta^2(\partial u_p/\partial r)^2(0.5\,(1-K_n)-(1-K_\tau)/7)^2}{6912\beta^2}$ (see Eqs. (33) and (34)). On the ascending portions of curves 1–6 the character of distribution of the solid phase dispersion energy components is determined by the considerable growth of the absolute value of the derivative $|\partial u_p/\partial r|$ (Fig. 2, curves 2, 4, 6). In the wall zone, the generation of the pseudoturbulent energy is decreased; therefore, the functions $\langle v'_p v'_p\rangle(r)$ and $\langle w'_p w'_p\rangle(r)$ decrease.

The profiles of the averaged and pulsating characteristics of a disperse flow are given in Fig. 4. It is seen that with an increase in the mean (over the section) gas velocity $u_{g,m}$ the character of the functions $\omega_\varphi(r)$ and $\langle\omega'_\varphi v'_p\rangle(r)$ does not change, but their absolute values somewhat increase (curves 2 and 3, 5 and 6 are compared).

Figure 5 demonstrates the distribution of the second and third moments of pulsations of the angular velocity of particles over the stretch of the stabilized two-phase flow. The behavior of the function $\langle\omega'_\varphi\omega'^2_r\rangle(r)$ (curve 1) within 0.06 m $< r <$ 0.1 m depends on the third, fourth, and sixth terms of Eq. (22). On the ascending branch, 0.075 m $< r <$ 0.093 m, the character of the $\langle\omega'_\varphi\omega'^2_r\rangle(r)$ curve is determined by the increase in the functions $\omega_\varphi(r)$, $\langle\omega'_\varphi\omega'_r w'_p\rangle(r)$, $\langle\omega'^2_\varphi\rangle(r)$, $\langle\omega'_r w'_p\rangle(r)$ and by the decrease of the $\langle\omega'^2_r v'_p\rangle(r)$ curve (Fig. 4, curves 2, 4, 7; Fig. 5, curve 2; Fig. 6, curve 5). Over the descending stretch, 0.093 m $< r <$ 0.0972 m, the decrease of the function $\langle\omega'_\varphi\omega'^2_r\rangle(r)$ is connected with the decrease of the third and sixth terms of the indicated equation due to the decrease of the curve $\langle\omega'_\varphi\omega'_r w'^2_p\rangle(r)$ and increase of the function $\langle\omega'^2_r v'_p\rangle(r)$. With a further increase of the radial coordinate ($r >$ 0.0972 m) the function $\langle\omega'_\varphi\omega'_r w'_p\rangle(r)$ becomes negative and its absolute value increases, as a result of which the $\langle\omega'_\varphi\omega'^2_r\rangle(r)$ curve continues to decrease in this region.



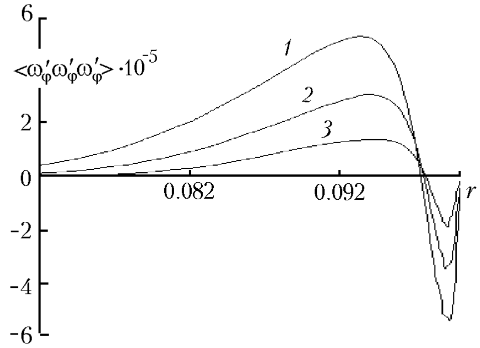


Fig. 6. Distribution of the second and third moments of pulsations of the linear and angular velocities of the dispersed phase: variant I — 1) $\langle \omega'_\varphi \omega'_r w'_p \rangle$; 2) $\langle \omega'^2_\varphi w'_p \rangle$; 3) $\langle \omega'^2_\varphi v'_p \rangle$; 4) $\langle \omega'_\varphi \omega'_r \rangle$; III — 5) $\langle \omega'_\varphi \omega'_r w'_p \rangle$.

Fig. 7. Distribution of the third moment of pulsations of the angular velocity of particles: 1) II; 2) III; 3) I.

It is seen from Fig. 5 that the function $\langle \omega'^2_\varphi \rangle(r)$ has a maximum at the point r = 0.0975 m (curve 2) the presence of which can be explained by the influence of the second, fourth, and eighth terms of Eq. (29). Within the range 0.073 m $< r <$ 0.0975 m the increase of the function $\langle \omega'^2_\varphi \rangle(r)$ is associated with the increase of the eighth term of the equation due to the rapid increase in the gradient of the angular velocity of particles $\partial\omega_\varphi/\partial r$ and decrease of the curve $\langle \omega'_\varphi v'_p \rangle(r)$ in this zone (Fig. 4, curves 2, 6). In this case the algebraic sum of the second, $\frac{\partial}{2r\partial r}\left(\frac{r\langle \omega'_\varphi v'_p \rangle \partial \langle \omega'_\varphi v'_p \rangle}{\psi_4 \partial r}\right) > 0$, and fourth, $\frac{\partial}{2r\partial r}\left(\frac{r\langle \omega'_\varphi v'^2_p \rangle \partial \omega_\varphi}{\psi_4 \partial r}\right) < 0$, terms of the equation is close to zero. Over the stretch with $r >$ 0.0975 m, where the second derivatives change signs and the value of the eighth term decrease, the second term starts to prevail over the fourth and the eighth ones, and therefore the function $\langle \omega'^2_\varphi \rangle(r)$ decreases in this region.

Figure 6 presents the results of calculations of the third-order correlation $\langle \omega'^2_\varphi v'_p \rangle$ over the stretch of the developed motion of gas suspension (curve 3). The balance of the terms of Eq. (18) shows that the basic role in the formation of the profile of $\langle \omega'^2_\varphi v'_p \rangle(r)$ is played by the first, second, and fourth terms of the equation named. The decrease of the curve $\langle \omega'^2_\varphi v'_p \rangle(r)$ in the range 0.070 m $< r <$ 0.091 m is associated with the increase of the functions $\langle v'^2_p \rangle(r)$, $\langle \omega'^2_\varphi \rangle(r)$, $\omega_\varphi(r)$, $\langle \omega'_\varphi v'^2_p \rangle(r)$ ($\langle \omega'_\varphi v'^2_p \rangle(r) > 0$) and the decrease of the function $\langle \omega'_\varphi v'_p \rangle(r)$ in this zone (Fig. 3, curve 2; Fig. 4, curves 3 and 5; Fig. 5, curve 4). Over the stretch 0.091 m $< r <$ 0.098 m the derivative $\partial\langle \omega'^2_\varphi \rangle/\partial r$ tends to zero and the function $\langle \omega'_\varphi v'^2_p \rangle(r)$ decreases, assuming negative values, as a result of which the function $\langle \omega'^2_\varphi v'_p \rangle(r)$ increases in this region. The further increase of the function $\langle \omega'^2_\varphi v'_p \rangle(r)$ within the range 0.098 m $< r <$ 0.099 m is associated with the increase of the values of $|\langle \omega'_\varphi v'^2_p \rangle|$ and $|\partial\langle \omega'^2_\varphi \rangle/\partial r|$. Near the channel wall the $\langle \omega'^2_\varphi v'_p \rangle(r)$ curve decreases due to the decrease in the normal Reynolds stress $\langle v'^2_p \rangle$, in the absolute values of the derivative $|\partial\langle \omega'^2_\varphi \rangle/\partial r|$, and in the correlation $|\langle \omega'_\varphi v'^2_p \rangle|$.

Figure 7 demonstrates the distribution of the third moment $\langle \omega'^3_\varphi \rangle$ over the cross section of the flow. An analysis of the results of calculations shows that the character of the function $\langle \omega'^3_\varphi \rangle(r)$ (curve 3) is formed under the influence of the first and second terms of Eq. (25). The monotonic increase in the function $\langle \omega'^3_\varphi \rangle(r)$ within the range $0 < r <$ 0.093 m is due to the decrease of the $\langle \omega'_\varphi v'_p \rangle(r)$ and $\langle \omega'^2_\varphi v'_p \rangle(r)$ curves and increase in the derivatives $\partial\langle \omega'^2_\varphi \rangle/\partial r(r)$ and $\partial\omega_\varphi/\partial r$ in that zone (Fig. 4, curves 3 and 5; Fig. 5, curve 4; Fig. 6, curve 3). On the descending branch with 0.093 m $< r <$ 0.099 m, two stretches can be distinguished. Over the first stretch with 0.093 m $< r <$ 0.0975 m the decrease in the function $\langle \omega'^3_\varphi \rangle(r)$ is associated with the decrease in the value of $|\langle \omega'^2_\varphi v'_p \rangle|$ and in the gradient $\partial\langle \omega'^2_\varphi \rangle/\partial r$. Over the second stretch with 0.0975 m $< r <$ 0.099 m, the function $\langle \omega'^2_\varphi v'_p \rangle(r)$ tends to zero. In this case the derivative $\partial\langle \omega'^2_\varphi \rangle/\partial r$ becomes negative, whereas its absolute value increases and as a result the $\langle \omega'^3_\varphi \rangle(r)$ curve continues to decrease in this interval. In the wall region, $r > 0.099$ m, the value of the third moment $\langle \omega_\varphi'^2 v_p' \rangle$ increases and the ratio $|\partial \langle \omega_\varphi'^2 \rangle / \partial r|$ decreases, as a result of which the function $\langle \omega_\varphi'^3 \rangle (r)$ increases over this stretch.

The above-described technique used to calculate the averaged and pulsating characteristics of a dispersed phase reflects the basic laws governing this complex class of two-phase flows.

NOTATION

C_1, C_2, empirical constants; F, force, kg/(sec^2·m^2); G, generation of turbulent energy of a gas in the wakes of particles, kg/(sec^3·m); g, free-fall acceleration, m/sec^2; K, coefficient of velocity recovery on impact; k, kinetic pulsation energy, m^2/sec^2; L_1, L_2, coefficients; N, frequency of impacts, 1/sec; P, gas pressure, N/m^2; R, radius of the channel, m; r, z, and φ, radial, longitudinal, and transversal coordinates, m; S, dissipation of the turbulent energy of gas due to the action of the force of interphase interaction, m^2/sec^4; u, v, w, averaged components of the velocity vector, m/sec; β, true volumetric concentration of particles; γ, coefficient, sec^{-1}; δ, diameter of a particle, m; ε, dissipation of pulsation energy, m^2/sec^3; ξ_1, ξ_2, χ, functions; η, kinematic viscosity, m^2/sec; λ, coefficient; ρ, density, kg/m^3; σ, empirical constant; τ, time of dynamic relaxation, sec; ψ_1, ψ_2, ψ_3, ψ_4, ψ_5, and ψ_6, coefficients, sec^{-1}; ω, angular velocity, sec^{-1}. Subscripts and superscripts: a, aerodynamic resistance of a particle; ax, longitudinal axis of the flow; d, dissipation; g, gas; m, mean (over the section); n, normal; p, particle; t, turbulent pulsations; w, channel wall; τ, tangential; $'$, pulsational component of averaging in time; $\langle\ \rangle$, averaging in time; ^, actual values.

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